The Stochastic Full Balance Sheet Model

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Surplus projections

1 in 10 1 yr surplus changes
Key questions in risk management

• What is the cost to our solvency position of a 20% equity fall?
• What fall in solvency is a 1in10 loss?
• What is the probability our capital coverage ratio goes below 120%?
• What is the probability we breach SCR?
• What event is most likely to cause a breach of SCR?
Risk appetite

- Risk appetite buffers take into account the stability of the capital position
Types of group model

Regulatory Capital

Economic Capital

Full Balance Sheet Model
Group model components

Model Structure

Risk Model

Loss Function

Group Model
Group model components
Regulatory capital – SII Internal Model

Model Structure
- Model structure part prescribed under SII

Risk Model
- Majority of firms use copula simulation models
- Individual distributions specified for each risk

Loss Function
- Losses to assets and liabilities normally estimated through proxy functions
Group model components

Regulatory capital – SII Standard Formula

Model Structure

- Model structure mostly prescribed under SII

Risk Model

- No model actually specified. May be thought of as multivariate normal

Loss Function

- Losses represented by linear loss functions fitted to the 1in200 points
- No cross terms to represent interaction between risks
Group model components

Economic capital

Model Structure

- Model structure not prescribed, however commonly similar to the regulatory capital model

Risk Model

- May be similar to a firm’s regulatory model
- Other risks could be included
- Different calibrations could be used.

Loss Function

- Commonly similar to the regulatory model
- Differences could be in:
  - MA or VA
  - Pension valuation
  - Contract boundaries
  - Etc.
Group model components
Stochastic full balance sheet model

- **Model Structure**
  - Structure may be close to economic capital model

- **Risk Model**
  - Represents a firm's best view of risks
  - Likely to be aligned to economic capital model

- **Loss Function**
  - Losses represent changes in the full SII balance sheet rather than just assets and liabilities
  - Need to allow for realistic changes in discount rates (VA, MA, IAS19)
Group model components
Stochastic full balance sheet model
Types of group model

- Regulatory Capital
- Full Balance Sheet Model

Strength of the Capital Position

Stability of the Capital Position
Simulation generation

• Simulation generation may use standard copula modelling techniques

We should consider

• Should risk calibrations be Point In Time or Through The Cycle?
Proxy models

- The purpose of a proxy model is to enable fast estimation of balance sheet changes as a function of risk movements.
The purpose of a proxy model is to enable fast estimation of balance sheet changes as a function of risk movements.
Roll forwards

- Roll forwards techniques are used to estimate how loss functions change
- Interest rate example,

\[ \text{Change in NAV} = X - 20X^2 \]

Say we have a 1.6% interest rate increase
New Change in NAV = \((X+1.6\%) - 20(X+1.6\%)^2\)
$$= 0.36X - 20X^2 + 0.01088$$
The solvency II balance sheet

- Transitional Measures
- Assets
- Risk Margin
- BEL
- SCR
- Surplus
Asset and liability modelling

- Asset and liability models typically the same as used in an Internal Model or Economic Capital model

Difficulties may arise over discount rates used for:

- Volatility Adjustment (VA) business
- Matching Adjustment (MA) business
- Pension liabilities
Example model
Annuity example - loss model

- 100,000 60 year old annuitants
- Annuity amount £1000 p.a.
- Expenses of £100, inflating at 1% p.a.
- Mortality as per an example mortality table
- Yield curve flat at 2%
- Risk free fixed interest cash-flows to broadly match the liability run off
Annuity example - risk model

- Normally distributed risks assumed for
  - Longevity
  - Expense
  - Inflation
  - Interest Rate PC1
  - Interest Rate PC2
  - Interest Rate PC3
  - Credit

- Risks aggregated using a Gaussian copula with specified correlations

<table>
<thead>
<tr>
<th></th>
<th>Longevity</th>
<th>Inflation</th>
<th>Expense</th>
<th>PC1</th>
<th>PC2</th>
<th>PC3</th>
<th>Credit</th>
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<tbody>
<tr>
<td>Longevity</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
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<td>0%</td>
<td>0%</td>
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<tr>
<td>Inflation</td>
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<td>100%</td>
<td>0%</td>
<td>50%</td>
<td>0%</td>
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<td>-20%</td>
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<tr>
<td>Expense</td>
<td>0%</td>
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<tr>
<td>PC1</td>
<td>0%</td>
<td>50%</td>
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<td>0%</td>
<td>-25%</td>
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<td>PC2</td>
<td>0%</td>
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<td>0%</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>Credit</td>
<td>0%</td>
<td>-20%</td>
<td>0%</td>
<td>-25%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
</tr>
</tbody>
</table>
Annuity example - principal components analysis

- PCA is a dimension reduction technique commonly used to model yield curve changes

![Diagram showing principal components analysis](image)

<table>
<thead>
<tr>
<th>Term1</th>
<th>Term2</th>
<th>Term3</th>
<th>Term4</th>
<th>Term5</th>
<th>Term6</th>
<th>Term7</th>
<th>Term8</th>
<th>Term9</th>
<th>Term10</th>
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<th>Term13</th>
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<th>Term15</th>
<th>Term16</th>
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<tbody>
<tr>
<td>PC1</td>
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</table>

![Change in Yield Graph](image)
Annuity example – fitting approach

- Use Sobol sequence of 1023 fitting points
- Polynomial terms up to order 4 used to fit function to net assets
- Step regression applied to fit the proxy functions
- Out Of Sample (OOS) testing carried out using 100 random points from the risk model.

Proxy change in NAV = \( f(L, l, E, PC1, PC2, PC3) \)
Annuity example – key exposures
Annuity example – joint exposure PC1 vs longevity

Longevity (change in qx)

Threat direction

PC1 Value
SCR Proxy Modelling
SCR proxy models – SF

1. Calibration runs
2. Generate proxy functions for each run
3. Use proxy function to get SF capitals
4. Roll Forward Process
5. Fit SCR proxy function
6. OOS testing

- Equity Interest
- Risk model

Change in NAV vs. Change in Interest Rates

Change in NAV:
- -2.0%
- 0.0%
- 2.0%

Change in Interest Rates:
- -0.04
- -0.03
- -0.02
- -0.01
- 0.00
- 0.01
- 0.02

Equity Interest
SCR proxy models – Standard Formula

- SCR Risk Model
  - Equity Risk Model
  - Interest Rate Risk Model
SCR proxy models – Standard Formula - longevity

- SF Longevity stress is a 20% fall in $q_x$

$SF \text{ Longevity Capital Estimate} = -f(-20\%,0,0,0,0,0)$
SCR proxy models – Standard Formula - expense

- SF Expense stress is a 10% increase in expenses, together with a 1% increase in expense inflation
- Our example model uses a separate expense level risk and inflation risk
- Estimate SF expense capital using a combined expense and inflation event

\[
SF \text{ Expense Capital Estimate} = f(0, 1\%, 10\%, 0, 0, 0)
\]
SCR proxy models – Standard Formula – interest rates

- SF Interest Rate up and down stresses are a function of the current yield curve
- We may estimate any change in yield curve as a linear combination of our principle components

Example, at yields of 2%, SF Yield down Capital Estimate = \(-f(0,0,0,-1.23,0.25,-1.16)\)
SCR proxy models – Standard Formula
SCR proxy models – Standard Formula – key exposures

- **Interest Rate PC1**
  - Change in NAV vs. PC1
  - Change in NAV: -40 to 40
  - PC1: -2.5 to 2.5

- **Longevity**
  - Change in NAV vs. qx stress
  - Change in NAV: -40 to 40
  - qx stress: -50% to 50%

- **Inflation**
  - Change in NAV vs. Inflation Stress
  - Change in NAV: -40 to 40
  - Inflation Stress: -2% to 2%

- **Expense**
  - Change in NAV vs. Expense stress
  - Change in NAV: -40 to 40
  - Expense stress: -50% to 50%
SCR proxy models – Internal Model

Calibration runs → Generate proxy functions for each run → Run SCR model for each run → Fit SCR proxy function → OOS testing → Roll Forward Process

Change in NAV vs. Change in Interest Rates

-0.04 → -0.03 → -0.02 → -0.01 → 0.00 → 0.01 → 0.02
SCR proxy models – Internal Model

[Image: Graph showing the relationship between IM SCR Proxy and Actual (GBP million)]
SCR proxy models – Internal Model key exposures

**Interest Rate PC1**

- NAV change vs. Interest Rate PC1
- Models: SF, IM

**Longevity**

- NAV change vs. qx stress
- Models: SF, IM

**Inflation**

- NAV change vs. Inflation stress
- Stress levels: -2%, -1%, 0%, 1%, 2%

**Expense**

- NAV change vs. Expense stress
- Stress levels: 50%, 30%, 10%, 0%, -10%, -30%, -50%
Risk Margin Proxy Modelling
Risk Margin proxy models

- Estimate changes in RM for calibration set
- Fit proxy function
- OOS testing for fit performance

Options:
- Changes in Discount rates
- No VA/MA in Risk Margin
- Changes in Run-off
Risk Margin proxy models – run off example

- Annuity BEL run off under stress
Risk Margin proxy models – Standard Formula

Risk Margin Proxy Fit

Proxy (£m) vs. Actual (£m)
Standard Formula example – key exposures

**Interest Rate PC1**

<table>
<thead>
<tr>
<th>NAV Change</th>
<th>-140</th>
<th>-90</th>
<th>-40</th>
<th>10</th>
<th>60</th>
<th>110</th>
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<tbody>
<tr>
<td>PC1</td>
<td></td>
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</table>

- Yellow line: Net Assets
- Blue line: Full Balance Sheet

**Longevity**

<table>
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<th>NAV Change</th>
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<td>qx stress</td>
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- Blue line: qx stress

**Inflation**

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- Blue line: Inflation stress

**Expense**

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- Blue line: Expense Stress

**Expense Stress**

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- Blue line: Expense Stress

**Expense**

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</table>

- Blue line: Expense
Discount Rates modelling
Liability modelling – Volatility Adjustment

- VA represents a flat addition to the discount curve for applicable long term liabilities
- Designed to protect insurers from the impact of volatility on the insurer’s solvency position
- Calculated as 65% of the spread between the interest rate of the assets in a reference portfolio and the risk free rate, allowing for a fundamental spread
- Published monthly by EIOPA
- Permitted to change under SCR stress by some European supervisors, more recently in the UK
Liability modelling – Volatility Adjustment

- How it works in practice

**Base**

| Assets | Liabilities | Surplus |

**Credit stress event**

| Assets | Liabilities | Surplus |
Liability modelling – Volatility Adjustment

• How it works under non-dynamic VA SCR calculations
Liability modelling – dynamic VA model

- VA ≈ 65% x (Spread – Fundamental Spread) calculated by rating, maturity
Liability modelling – dynamic VA model
Liability modelling

- We need realistic models to take into account the movement of these under stress.

Diagram showing the relationship between IAS19 discount rates and AA yield.
Include credit risk as an additional normally distributed risk in the model.
Annuity example - dynamic VA, SF model

Repeat curve fit process for net assets and SCR, RM unchanged.
Annuity example - dynamic VA, SF model

Low materiality changes in other risk exposures
Using the Example Model
More assumptions

• Starting surplus = £250m
• Risk Appetite thresholds (based on a one year time frame):
  – Plan to be able to withstand a 1in30 shock
  – We take urgent action if our surplus is unable to withstand a 1in10 shock
Risk appetite

![Risk appetite graph]

- Change in NAV (£m)
- Density

Risk appetite categories:
- Normal
- Improve
- Take Action
Ruin probabilities

Ruin Prob = 5.9%
Euler allocation by risk – ruin event of £250m loss

- Euler Allocation of 250m loss

Risk A allocation = \(-E[X_A | X_{total} = -250m]\)
Ruin Events
Euler example

- Risks A and B, multivariate standard normal
- Correlation -99.9%

\[
\text{Change in NAV} = 2 - (e^A + e^B)
\]

1 in 200 capital = 14.8m

Euler allocations:
- Risk A = 7.4m
- Risk B = 7.4m
Ruin events – Most Likely Ruin Event

![Graph showing the relationship between density and risk movement with a peak indicating the Most Likely Ruin Event (MLRE) and a shaded area representing the ruin region.](image-url)
Ruin events – Most Likely Ruin Event

Ruin region

MLRE
Ruin events – Euler example

- Risk distribution is multivariate normal (-99.9% correlation)
- Density function of A, B is well known \( f(A,B) \)
- Can solve for the maximum of \( f(A,B) \) subject to constraint \( 2 - (e^A + e^B) = -14.8 \)

Max at \((A, B) = (-2.8, 2.8) \) and \((2.8, -2.8) \)
Ruin events – Euler example

Ruin region

A = -B

Most likely points

A

B
Ruin events – annuity model

- Find MLRE by maximising probability density subject to change in NAV < -£250m
- Risk distribution ~ Multivariate normal so density is well defined
- Change in NAV estimated using proxy functions.

<table>
<thead>
<tr>
<th>Risk</th>
<th>Movement</th>
<th>Percentile</th>
<th>1 in X</th>
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<tbody>
<tr>
<td>PC1</td>
<td>0.437</td>
<td>0.67</td>
<td>3.0</td>
</tr>
<tr>
<td>PC2</td>
<td>-0.209</td>
<td>0.42</td>
<td>2.4</td>
</tr>
<tr>
<td>PC3</td>
<td>-0.128</td>
<td>0.45</td>
<td>2.2</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.6%</td>
<td>0.78</td>
<td>4.6</td>
</tr>
<tr>
<td>Credit (spreads)</td>
<td>-1.8%</td>
<td>0.12</td>
<td>8.1</td>
</tr>
<tr>
<td>Longevity ($q_x$)</td>
<td>-3.8%</td>
<td>0.31</td>
<td>3.2</td>
</tr>
<tr>
<td>Expense (level)</td>
<td>14.1%</td>
<td>0.77</td>
<td>3.0</td>
</tr>
</tbody>
</table>

![Yield stress graph](image)
Ruin events – annuity model

Risk as % of 1in10

- Expense
- Longevity
- Credit
- Inflation
- PCB
- PC2
- PC1

Cost (£m)

Loss by Risk

PC1, PC2, PC3, Inflation, Credit, Longevity, Expense, Cross terms
Ruin events – Kernel Density Estimation

- We can use Kernel Density Estimation (KDE) to estimate the density function of the joint risk distribution from the simulations
Ruin events – annuity model

Loss by risk – Direct vs KDE

- Direct
- KDE
Ruin cause

Ruin primary cause

<table>
<thead>
<tr>
<th>No.</th>
<th>Event</th>
<th>%</th>
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<tbody>
<tr>
<td>1</td>
<td>Credit</td>
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<tr>
<td>2</td>
<td>Credit / Inflation</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>Inflation</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>Expense</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>Credit / Expense</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Others</td>
<td>11</td>
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</table>
## Ruin cause

<table>
<thead>
<tr>
<th>No.</th>
<th>Event</th>
<th>Credit (change in spreads)</th>
<th>Inflation (change in RPI)</th>
<th>Expense (change in level)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Credit</td>
<td>-3.5%</td>
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</tr>
<tr>
<td>2</td>
<td>Credit / Inflation</td>
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<td>0.9%</td>
<td></td>
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<td>3</td>
<td>Inflation</td>
<td></td>
<td>2.8%</td>
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<tr>
<td>4</td>
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<td></td>
<td>68.7%</td>
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<tr>
<td>5</td>
<td>Credit / Expense</td>
<td>-2.5%</td>
<td></td>
<td>19.6%</td>
</tr>
</tbody>
</table>
Ruin events summary

- Euler is about capital allocation not events
- MLREs give insight into actual events
- KDE can be used to get the density from the simulations
- We can plan what we would do under the ruin events
Uses of the model
The roll forward cycle

Roll Forwards for:

- Run off
- New Business
- Economics
- Basis Changes
- Model changes
- Risk calibration changes

Assess Roll forwards performance → Recalibrate → Roll Forwards
Projections

- Is our balance sheet getting more or less stable over time?
- Is our ruin probability getting better or worse?
Projections

• Proxy functions normally express changes in NAV as a function of risk movements

\[ \text{Proxy change in NAV} = f(L, l, E, PC1, PC2, PC3) \]

• For projections (e.g. of risk appetite), we need to calibrate as a function of risks and time

\[ \text{Proxy change in NAV} = f(L, l, E, PC1, PC2, PC3, \text{Time}) \]

• We can use run off drivers by risk and product to scale the proxy functions over time
Projections - example

- For Risks X and Y

Proxy change in NAV = aX² + bX + cY + d XY
What isn’t in your model?

- Regulatory risk?
- Changes to business plan?
- Regime changes?
- Long term risks?
- Liquidity risk?
Types of group model

- Regulatory Capital
- Economic Capital
- Full Balance Sheet Model
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